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Hypothesis Tests of Goodness-of-Fit for Fréchet Distribution

Abidin, N. Z.*, Adam, M. B. and Midi, H.

Institute for Mathematical Research, Universiti Putra Malaysia, 43400 Serdang, Selangor, Malaysia

ABSTRACT

Extreme Value Theory (EVT) is a statistical field whose main focus is to investigate extreme phenomena. In EVT, Fréchet distribution is one of the extreme value distributions and it is used to model extreme events. The degree of fit between the model and the observed values was measured by Goodness-of-fit (GOF) test. Several types of GOF tests were also compared. The tests involved were Anderson-Darling (AD), Cramer-von Mises (CVM), Zhang Anderson Darling (ZAD), Zhang Cramer von-Mises (ZCVM) and L_n. The values of parameters μ, σ and ξ were estimated by Maximum Likelihood. The critical values were developed by Monte-Carlo simulation. In power study, the reliability of critical values was determined. Besides, it is of interest to identify which GOF test is superior to the other tests for Fréchet distribution. Thus, the comparisons of rejection rates were observed at different significance levels, as well as different sample sizes, based on several alternative distributions. Overall, given by Maximum Likelihood Estimation of Fréchet distribution, the ZAD and ZCVM tests are the most powerful tests for smaller sample size (ZAD for significance levels 0.05 and 0.1, ZCVM for significance level 0.01) as compared to AD, which is more powerful for larger sample size.

Keywords: Critical values Fréchet distribution, goodness-of-fit, rejection rate

INTRODUCTION

Extreme Value Theory (EVT) is a statistical discipline with the main focus is to estimate the probability of the occurrence of extreme phenomenon. The extreme event occurs either at maximum or minimum level (Coles, 2001). For instance, the likelihood of the flood during certain period of time is predicted by observing the volume of water at maximum level. Likewise, the study on the occurrence of draught was done by taking into account the minimum volume of rainfall (Castillo *et al*., 2005). These maximum and minimum values were collected and modelled based on the statistical extreme models.

There are several statistical extreme models in EVT that are broadly used for the extrapolation. The EVT comprises Gumbel,

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Email addresses:
Abidin, N. Z.(nahdiya@upm.edu.my), Adam, M. B. (bakri@upm.edu.my), Midi, H. (habshah@upm.edu.my)
*Corresponding Author

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Weibull and Fréchet's models. A combination of the three extreme models by Jenkinson (1955) is known as Generalized Extreme Value (GEV). The combination was introduced for the purpose of simplifying the modelling steps. In other words, instead of testing several assumptions on which extreme model is more likely to fit the data, GEV allows for the determination of the most suitable model based on a single value of shape parameter, X , which represents the types of tail behaviour (Coles, 2001). Each kind of extreme models has certain interval value of ξ explained in Theorem 1.1. For the maximum level of extreme, the extreme behaviour is denoted as $M_n = \max(X_1, ..., X_n)$, where M_n is the maximum value of the observation X over n time.

Theorem 1.1: *Let $X_1, ..., X_n$ be an independent random variables with the distribution F, and let asymptotic argument be $M_n = \max(X_1, ..., X_n)$. As the constants $a_n > 0$ and b_n exist, denote*

$$\lim_{n \to \infty} \Pr\left(\frac{M_n - b_n}{a_n} \leq x\right) \to F(x) \quad [1]$$

If the non-degenerate function F exists, then GEV is:

$$F(x) = \exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]_+^{-\frac{1}{\xi}}\right\} \quad [2]$$

where $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$ are the location, scale and shape parameters respectively. The GEV distribution should comply with $\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right] > 0$. The distribution of Gumbel, Fréchet, and Weibull corresponds to $\xi = 0$, $\xi > 0$, and $\xi < 0$, respectively.

Given that the main concern of EVT is to model the extreme behaviour, it is crucial that the extreme model is able to reflect the real event. This is because the choice of the extreme model affects the outcomes of decision making and problem solving (Castillo *et al.*, 2005). Therefore, a careful model validation is necessary. The validation test is called Goodness-of-fit (GOF) test. GOF test is a statistical test used to measure the fit of the selected statistical model against the observed values (Kinnison, 1989). When the model fits the observed distribution, the model can be utilized to interpret the behaviour of the distribution as well as to predict the outcomes. Thus, the GOF test plays a significant role to ensure the selected model is able to precisely reflect the population of the observed values (Shabri & Jemain, 2008). The GOF test that is commonly used is the hypothesis test using empirical distribution function (Zempléni, 2004). Several classical GOF tests are Anderson Darling (AD), Cramer-von Mises (CVM) and Kolmogrov Smirnov (KS). The performances of the GOF tests vary, depending on the types of the distribution and the methods of parameter estimation. Hence, in order to select the most appropriate GOF test, studies have been done to examine the power of the GOF tests for certain statistical distributions.

For extreme circumstances, there have been many studies conducted on the performance of the GOF tests for Gumbel and Weibull distributions (Kimber, 1985; Coles, 1989; Kinnison, 1989; Lockhart & Stephens, 1994; Liao & Shimokawa, 1999; Kotz & Nadarajah, 2000; Shabri

& Jemain, 2009; Laio, 2004; Zempleni, 2004). Other than that, Zhang (2002) and Zhang and Wu (2005) modified the classical AD, CVM and KS tests. These modified tests are named after Zhang who is also known as Zhang Anderson Darling (ZAD), Zhang Cramer-von Mises (ZCVM) and Zhang Kolmogorov Smirnov (ZKS). These later tests are more powerful than the classical tests, except for ZKS. However, the Zhang tests were evaluated for Normal distribution. Therefore, it is of interest to test the power of the Zhang test for the GEV distribution, particularly for Fréchet distribution. This is because the assessment of the GOF test for Fréchet distribution has yet to receive extensive attention.

Although only a few studies on the GOF test for Fréchet distribution have ever been discussed (Koning & Peng, 2008; Abd-Elfattah *et al*., 2010), this distribution plays a major role in the modelling of extreme events. Fréchet has been used to model heavy tailed distribution such as the option pricing in the extreme financial losses (Markose & Alentorn, 2011), hydrology and internet traffic (Koning & Peng, 2008). Moreover, many practical problems have the limit of maxima values that converge to Fréchet distribution (Castillo *et al.*, 2005). Thus, the identification of the best GOF test for Fréchet is important to facilitate the practitioners to have more precise evaluation on the degree of fit between the model and the observed values so that more reliable prediction on the extreme events related to Fréchet can be achieved. Therefore, the purpose of this study is to identify the most powerful GOF test coupled with parameter estimate of Maximum Likelihood for Fréchet distribution.

METHODOLOGY

The hypotheses for the GOF test are shown below. The proposition that the hypothetical statistical model fits the observed distribution is equivalent to fail to reject H_0. Otherwise, the rejection of H_0 implies the model does not fit the observed values. In this study, the hypothetical distribution is the Fréchet distribution.

H_0: The hypothetical model fits the observed distribution or $F_h(x) = F(x)$
H_1: The hypothetical model does not fit the observed distribution or $F_h(x) \neq F(x)$

Critical Values of the GOF Tests

Critical values are the baseline of whether to reject or fail to reject H_0. If H_0 is to be rejected, the statistics value produced by the GOF test should be able to exceed the critical value. The development of critical values was made by means of Monte Carlo simulation:

Step (A): Generate the random variables for sample of size 15. Random variables were generated from the inverse function of GEV with parameters μ = 100, σ = 10 and ξ = 0.1, 0.2, 0.3, 0.4, 0.5. The values of ξ are within the values exhibited by Shin *et al* (2012), Shabri, and Jemain (2008), and Ahmad (1988). Moreover, standard values for μ and σ are 0 and 1, respectively (Shin *et al*., 2012; Shabri & Jemain, 2008; Ahmad, 1988). However, the value of σ = 10 was selected for simulation because the dispersion is wider than the standard value. Hence, it is easier to reach the convergence of parameter estimation using Maximum

Likelihood Estimation, especially for small sample size. Furthermore, to determine whether the critical values of μ = 100 and σ = 10 are similar to the standard values, the rejection rate for Fréchet with μ = 0, σ = 1 was compared. The inverse function of GEV is:

$$F(x)^{-1} = \mu - \frac{\sigma}{\xi}(1 - \log U)^{-\xi} \quad [3]$$

where U is hypothetical distribution function. The value for U is $U_{i,n} = \frac{i - 0.5}{n}$. These random variables were arranged in the ascending order.

Step (B): The parameters were estimated. The values of parameters μ, σ and ξ were estimated by Maximum Likelihood Estimation. The loglikelihood of Fréchet distribution is

$$l(\mu,\sigma,\xi) = -\sum_{i=1}^{n}\left[1 + \xi\left(\frac{x_i - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi}} - \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n}\log\left[1 + \xi\left(\frac{x_i - \mu}{\sigma}\right)\right] - n\log(\sigma) \quad [4]$$

The maximum likelihood estimate is obtained by maximizing the loglikelihood expression by way of partial derivatives method. These differentiations were carried out with respect to each parameter.

Step (C): Based on the arranged random variables and the estimated parameters, the cumulative distribution function, *F(x),* for Fréchet was determined.

Step (D): The values of *F(x)* were substituted into the expressions of the GOF tests. The expression of the GOF tests involved is shown below. The values produced by the expression of the GOF tests are called the statistics values:

Anderson Darling test (AD):

$$AD = \sum_{i=1}^{n}\frac{2i - 1}{n}\{\log[F(x_i)] + \log[1 - F(x_{n+1-i})]\} - n \quad [5]$$

Cramer von-Mises test (CVM):

$$CVM = \frac{1}{12n} + \sum_{i=1}^{n}\left[F(x_i) - \frac{2i - 1}{2n}\right]^2 \quad [6]$$

Zhang Anderson Darling test (ZAD):

$$ZAD = \sum_{i=1}^{n}\left\{\frac{\log[F(x_i)]}{n - i + 0.5} + \frac{\log[1 - F(x_i)]}{i - 0.5}\right\} \quad [7]$$

Zhang Cramer von-Mises test (ZCVM):

$$ZCVM = \sum_{i=1}^{n} \left\{ \log \left[\frac{\frac{1}{F(x_i)} - 1}{\frac{n-0.5}{i-0.75} - 1} \right] \right\}^2 \qquad [8]$$

L_n test:

$$L_n = \frac{1}{\sqrt{n}} \frac{\max\left[\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n}\right]}{\sqrt{F(x_i)[1-F(x_i)]}} \qquad [9]$$

Step (E): The steps from A to D were iterated for 10,000 times. The 10,000 iterations yield 10,000 statistics values. Those 10,000 statistics values were arranged in the ascending order. The order of the statistics value at the percentiles of 99, 95, and 90 implies the critical value at the significance level of 0.01, 0.05 and 0.10, respectively.

The association between different values of ξ and the critical values was performed by means of average. In other words, the critical values obtained from $\xi = 0.1, 0.2, 0.3, 0.4, 0.5$ were averaged for each sample size and significance level. Thus, for $0.1 < \xi < 0.5$, there is a single critical value at a particular sample size, as well as significance level. The steps from A to E were done for a sample of size n=20, 30, 40, 50 and 100.

Power of the GOF Tests

Power is the statistics value of $F(x)$ that exceeds the critical value of $F_h(x)$. Similarly, it is also known as the probability of rejecting H_0. Low rejection rate implies low probability of rejecting H_0. Hence, the rejection rate is normally applied to check the probability of failing to reject H_0. When $F_h(x) = F(x)$ is true, the rejection rate approximates the respective significance level (Laio, 2004; Shin *et al.,* 2012). The ability of any GOF test to get these approximate values signifies that the respective critical values obtained are reliable. The assessment is done by comparing the hypothetical distribution which is Fréchet with other observed values from Fréchet distributions, which are: Fréchet with true parameters values ($\mu = 100$, $\sigma = 10$ and $\xi = 0.5$) and Fréchet with different parameter values ($\mu = 0$, $\sigma = 1$ and $\xi = 0.3$).

In contrast, high rejection rate implies a high probability of rejecting H_0. When $F_h(x) \neq F(x)$, the rejection rate is higher than the respective significance level (Laio, 2004; Shin *et al.,* 2012). The higher the rejection rate, the more powerful the GOF test is. Therefore, rejection rate is also used to evaluate the degree of power of the GOF test. This evaluation is done by comparing the Fréchet model with the alternative statistical distributions. Given by the alternative distributions, the power test was conducted to evaluate the ability of the GOF tests to exceed the critical values of Fréchet and subsequently reject H_0.

Meanwhile, the Monte Carlo method is used to assess the degree of rejection rate of the GOF tests using different sample size and at different significance levels. The steps of calculating the rejection rate are similar to the steps done for critical values mentioned previously. For step A, the random variables of Fréchet with true parameter values $\mu = 100$, $\sigma = 10$ and $\xi = 0.5$ were simulated. In addition, the random variables of Fréchet for different parameter values $\mu = 0$, $\sigma = 1$ and $\xi = 0.3$ were also generated. After that, the random variables of the following alternative distributions were simulated: Gamma distribution; Gamma~(3,1), Weibull distribution; weibull~(0,1,0.5), Lognormal distribution; Lognorm~(0, 1, 0.5), Normal distribution; N~(0,1), Generalized Logistic distribution; Glog~(0, 1, -0.5), and Exponential distribution; Exp~(1).

Next, the steps from B to D were followed orderly. At step E, the simulations were done for 10,000 iterations. The power values of GOF tests were determined by averaging the 10,000 statistics values exceeding the critical values of Fréchet.

RESULTS

Table 1 represents the critical values of the GOF tests for Fréchet distribution with respect to the sample of sizes, *n*= 15, 20, 30, 40 50 and 100. These critical values of the GOF tests for the Fréchet distribution are illustrated in Fig.1. In Fig.1, the critical values of AD is on the top left, ZAD is on the top right, CVM is on the middle left, ZCVM is on the middle right and L_n is on the bottom left. Based on Table 1 and Fig.1, the critical values of AD are within the interval of 0.4 to 0.9. On the other hand, the critical values of ZAD are around 3.4 as the sample size increases. The critical values for CVM ranged from 0.05 to 0.15. For ZCVM and L_n, the critical values increase along with the increment of sample size, starting from 4 to 16 and 2 to 2.8, respectively.

The reliability of the critical values was observed from the rejection rates for Fréchet ($\mu = 100$, $\sigma = 10$ and $\xi = 0.5$) and Fréchet ($\mu = 0$, $\sigma = 1$ and $\xi = 0.3$), as presented in Table 2. Table 2 shows that the rejection rates of the GOF tests for both the distributions are close to the respective significance level. The rejection rates of the GOF tests at the significance level of 0.01 are approximately 0.01. The same trend goes to the rejection rates at 0.05 and 0.1 significance levels. Moreover, Fig.2 (top left and top right) portrays the rejection rates at significance level of 0.05. The rejection rates of all the GOF tests are within 0.04 and 0.06.

In addition, Table 2 exhibits the rejection rates of the GOF tests for the alternative distributions. The values highlighted in bol represent the highest rejection rate at each sample size, significance level and alternative distribution. The results show that at 0.01 significance level, ZCVM is the most powerful test for small sample sizes, which are *n*=15 and 20. For *n*=30, ZAD outperforms the other GOF tests. For larger sample size (*n*=40, 50 and 100), AD test is superior to other GOF tests. On the other hand, the results of the rejection rates at 0.05 and 0.1 significance levels are similar. For *n*=15, 20, 30 and 40, the most powerful test is ZAD. For *n*=50 and 100, ShabriJemain AD produces the greatest rejection rates than other competitors. The results for the rejection rates at significance level of 0.05 are depicted by Figures 2 and 3.

TABLE 1
Critical values of the GOF tests for Fréchet distribution

Test	AD			ZAD		
sig.lvl / n	0.10	0.05	0.01	0.10	0.05	0.01
15	0.457	0.546	0.800	3.355	3.380	3.470
20	0.467	0.534	0.716	3.350	3.371	3.418
30	0.476	0.554	0.771	3.345	3.363	3.402
40	0.479	0.560	0.765	3.340	3.353	3.387
50	0.481	0.577	0.769	3.336	3.348	3.374
100	0.496	0.583	0.782	3.323	3.330	3.350
Test	CVM			ZCVM		
sig.lvl / n	0.10	0.05	0.01	0.10	0.05	0.01
15	0.076	0.092	0.133	4.648	5.455	7.888
20	0.077	0.091	0.125	5.231	6.076	7.905
30	0.078	0.095	0.132	6.275	7.356	9.579
40	0.079	0.093	0.133	6.943	8.206	10.745
50	0.079	0.096	0.130	7.598	8.755	11.780
100	0.082	0.098	0.133	9.600	11.176	15.057
Test	L_n					
sig.lvl / n	0.10	0.05	0.01			
15	2.037	2.216	2.671			
20	2.088	2.265	2.641			
30	2.155	2.345	2.728			
40	2.202	2.411	2.803			
50	2.220	2.417	2.804			
100	2.296	2.482	2.871			

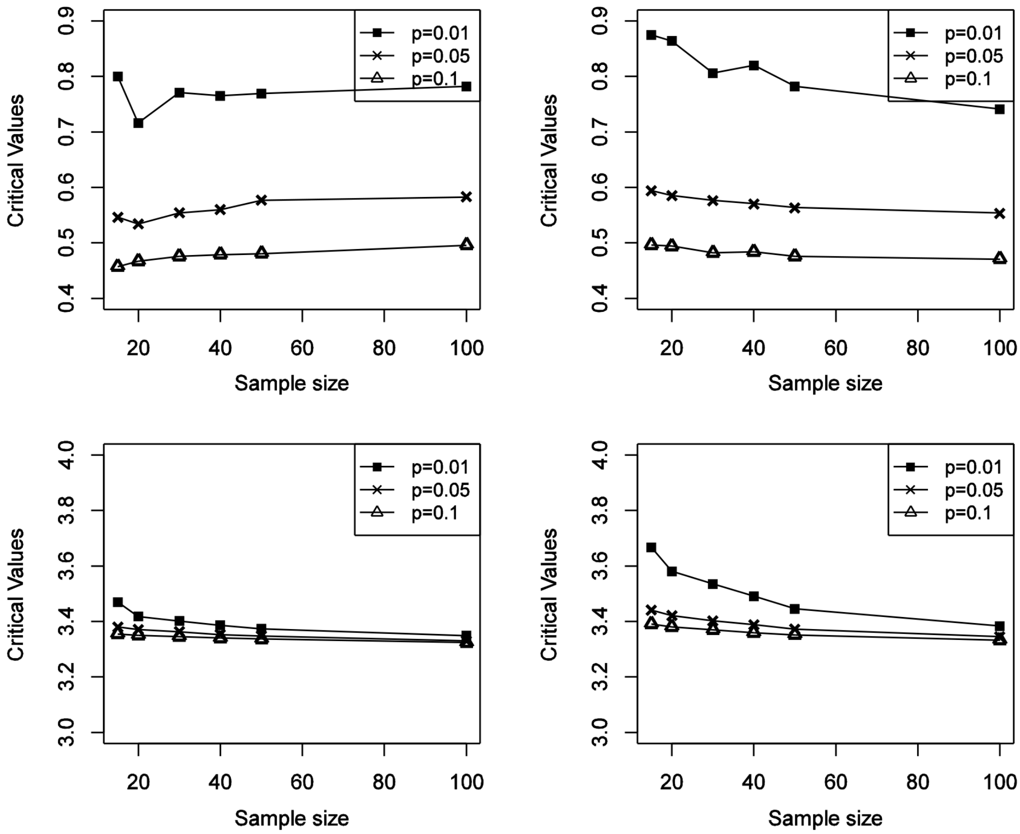

Fig.1: Critical values of the GOF tests for Fréchet distribution with respect to the sample of size, n= 15, 20, 30, 40 50 and 100. The critical values of AD is on the top left, ZAD is on the top right, CVM is on the middle left, ZCVM is on the middle right and L_n is on the bottom left.

TABLE 2
Rejection rate for GOF tests based on the selected distributions other than Fréchet

Distribution	Test	AD			ZAD			CVM		
	sig. lvl / n	0.10	0.05	0.01	0.10	0.05	0.01	0.10	0.05	0.01
GEV type-II (μ = 100, σ = 10 and ξ = 0.5)	15	0.100	0.054	0.012	0.093	0.056	0.012	0.104	0.054	0.013
	20	0.103	0.051	0.010	0.113	0.050	0.018	0.097	0.050	0.011
	30	0.086	0.052	0.009	0.103	0.049	0.010	0.095	0.046	0.012
	40	0.101	0.050	0.010	0.101	0.047	0.011	0.107	0.055	0.008
	50	0.103	0.050	0.014	0.101	0.048	0.012	0.097	0.054	0.015
	100	0.093	0.049	0.014	0.091	0.046	0.005	0.091	0.051	0.013

TABLE 2 (continue)

GEV type-II ($\mu = 0$, $\sigma = 1$ and $\xi = 0.3$)	15	0.109	0.055	0.011	0.118	0.059	0.014	0.113	0.057	0.011
	20	0.099	0.054	0.016	0.110	0.058	0.016	0.097	0.055	0.011
	30	0.097	0.051	0.009	0.099	0.052	0.019	0.090	0.044	0.008
	40	0.108	0.045	0.010	0.118	0.052	0.012	0.113	0.049	0.010
	50	0.106	0.045	0.012	0.111	0.054	0.015	0.115	0.054	0.013
	100	0.092	0.053	0.012	0.114	0.057	0.010	0.088	0.054	0.012
Gamma	15	0.811	0.695	0.460	**0.872**	**0.770**	0.500	0.752	0.625	0.438
	20	0.783	0.681	0.547	**0.834**	**0.766**	**0.600**	0.708	0.586	0.472
	30	0.786	0.674	0.472	**0.828**	**0.689**	**0.486**	0.734	0.605	0.421
	40	0.761	0.663	**0.511**	**0.778**	**0.675**	0.472	0.682	0.608	0.468
	50	0.769	**0.687**	**0.476**	0.750	0.661	0.457	0.715	0.620	0.418
	100	**0.743**	**0.674**	**0.497**	0.674	0.586	0.362	0.694	0.607	0.464
Weibull	15	0.793	0.717	0.470	**0.837**	**0.801**	0.506	0.733	0.651	0.437
	20	0.786	0.714	0.534	**0.845**	**0.758**	0.554	0.714	0.640	0.474
	30	0.781	0.684	0.494	**0.817**	**0.713**	0.482	0.698	0.601	0.455
	40	**0.776**	**0.686**	**0.485**	0.766	0.658	0.483	0.731	0.613	0.417
	50	**0.769**	**0.685**	**0.486**	0.761	0.667	0.452	0.695	0.616	0.440
	100	**0.723**	**0.656**	**0.509**	0.674	0.580	0.348	0.649	0.589	0.437
Generalized Logistic	15	0.818	0.692	0.467	**0.870**	**0.769**	0.514	0.746	0.624	0.434
	20	0.773	0.733	0.556	**0.831**	**0.788**	0.582	0.700	0.654	0.494
	30	0.767	0.669	0.487	**0.797**	**0.689**	**0.502**	0.691	0.591	0.458
	40	0.756	0.687	**0.497**	**0.804**	**0.697**	0.461	0.696	0.631	0.437
	50	0.775	**0.669**	**0.506**	**0.776**	0.663	0.476	0.709	0.612	0.443
	100	**0.758**	**0.681**	**0.532**	0.682	0.588	0.350	0.664	0.601	0.473
Exponential	15	0.801	0.718	0.477	**0.865**	**0.773**	0.503	0.729	0.655	0.453
	20	0.767	0.728	0.554	**0.835**	**0.796**	**0.587**	0.722	0.643	0.490
	30	0.776	0.701	0.487	**0.796**	**0.711**	0.503	0.690	0.635	0.442
	40	**0.785**	0.677	**0.505**	0.775	**0.682**	0.469	0.733	0.612	0.440
	50	0.789	0.672	0.504	0.769	0.656	0.465	0.709	0.610	0.458
	100	**0.749**	**0.675**	**0.499**	0.685	0.583	0.361	0.681	0.624	0.442
Lognormal	15	0.769	0.697	0.476	**0.833**	**0.768**	0.520	0.687	0.627	0.445
	20	0.796	0.720	0.563	**0.846**	**0.753**	0.587	0.705	0.651	0.478
	30	0.793	0.680	0.517	**0.832**	**0.712**	**0.529**	0.735	0.593	0.458
	40	0.785	0.672	**0.500**	**0.790**	**0.693**	0.482	0.728	0.611	0.434
	50	**0.786**	**0.700**	**0.504**	0.778	0.659	0.480	0.711	0.636	0.459
	100	**0.771**	**0.647**	**0.489**	0.702	0.587	0.364	0.701	0.585	0.443
Normal	15	0.793	0.696	0.466	**0.845**	**0.771**	0.495	0.727	0.639	0.434
	20	0.783	0.719	0.535	**0.848**	**0.747**	0.605	0.713	0.656	0.492
	30	0.750	0.708	0.511	**0.806**	**0.719**	**0.540**	0.678	0.629	0.473
	40	0.738	0.681	**0.500**	**0.790**	**0.704**	0.492	0.669	0.619	0.438
	50	**0.785**	**0.651**	**0.500**	0.775	0.626	0.483	0.716	0.605	0.455
	100	**0.775**	**0.641**	**0.490**	0.718	0.581	0.353	0.703	0.586	0.444

TABLE 2 (continue)

Distribution	Test: sig. lvl / n	ZCVM 0.10	ZCVM 0.05	ZCVM 0.01	L_n 0.10	L_n 0.05	L_n 0.01
GEV type-II ($\mu = 100$, $\sigma = 10$ and $\xi = 0.5$)	15	0.099	0.054	0.013	0.106	0.051	0.015
	20	0.099	0.049	0.013	0.107	0.043	0.013
	30	0.096	0.057	0.013	0.097	0.049	0.010
	40	0.110	0.043	0.010	0.099	0.048	0.014
	50	0.099	0.056	0.008	0.095	0.047	0.016
	100	0.100	0.046	0.008	0.094	0.057	0.007
GEV type-II ($\mu = 0$, $\sigma = 1$ and $\xi = 0.3$)	15	0.118	0.049	0.012	0.113	0.057	0.014
	20	0.116	0.058	0.015	0.091	0.058	0.018
	30	0.114	0.049	0.017	0.093	0.055	0.014
	40	0.119	0.055	0.006	0.109	0.048	0.011
	50	0.117	0.049	0.017	0.105	0.052	0.012
	100	0.107	0.057	0.008	0.104	0.052	0.015
Gamma	15	0.854	0.759	**0.532**	0.650	0.558	0.371
	20	0.801	0.736	**0.600**	0.634	0.531	0.421
	30	0.795	0.675	0.480	0.658	0.535	0.359
	40	0.762	0.648	0.472	0.611	0.503	0.347
	50	0.728	0.664	0.423	0.625	0.547	0.341
	100	0.664	0.566	0.353	0.624	0.542	0.373
Weibull	15	0.816	0.776	**0.537**	0.656	0.569	0.347
	20	0.819	0.752	**0.570**	0.658	0.555	0.382
	30	0.785	0.690	**0.499**	0.628	0.528	0.371
	40	0.756	0.626	0.464	0.653	0.492	0.323
	50	0.750	0.659	0.419	0.618	0.531	0.339
	100	0.649	0.562	0.334	0.572	0.499	0.345
Generalized Logistic	15	0.859	0.763	**0.527**	0.684	0.550	0.363
	20	0.817	0.758	**0.583**	0.638	0.564	0.390
	30	0.765	0.660	0.495	0.640	0.511	0.358
	40	0.785	0.664	0.454	0.610	0.520	0.344
	50	0.754	0.665	0.454	0.631	0.532	0.355
	100	0.661	0.564	0.349	0.603	0.503	0.374
Exponential	15	0.840	0.756	**0.525**	0.645	0.567	0.359
	20	0.808	0.780	**0.587**	0.641	0.573	0.394
	30	0.766	0.679	**0.504**	0.635	0.541	0.342
	40	0.765	0.664	0.477	0.646	0.509	0.366
	50	0.741	0.644	0.440	0.646	0.537	0.364
	100	0.677	0.564	0.360	0.615	0.528	0.364

TABLE 2 (continue)

Lognormal	15	0.813	0.758	**0.535**	0.634	0.538	0.358
	20	0.814	**0.753**	**0.589**	0.660	0.562	0.398
	30	0.776	0.683	0.527	0.660	0.533	0.405
	40	0.768	0.654	0.481	0.645	0.497	0.330
	50	0.748	0.651	0.441	0.627	0.547	0.350
	100	0.690	0.565	0.350	0.629	0.503	0.353
Normal	15	0.835	0.747	**0.520**	0.636	0.569	0.364
	20	0.821	0.736	**0.619**	0.644	0.581	0.382
	30	0.775	0.695	0.531	0.604	0.553	0.376
	40	0.769	0.672	0.479	0.595	0.492	0.338
	50	0.755	0.627	0.448	0.656	0.516	0.362
	100	0.693	0.551	0.348	0.631	0.518	0.364

* The value in **bold** denotes the highest rejection rate for each sample size, significance level and alternative distribution.

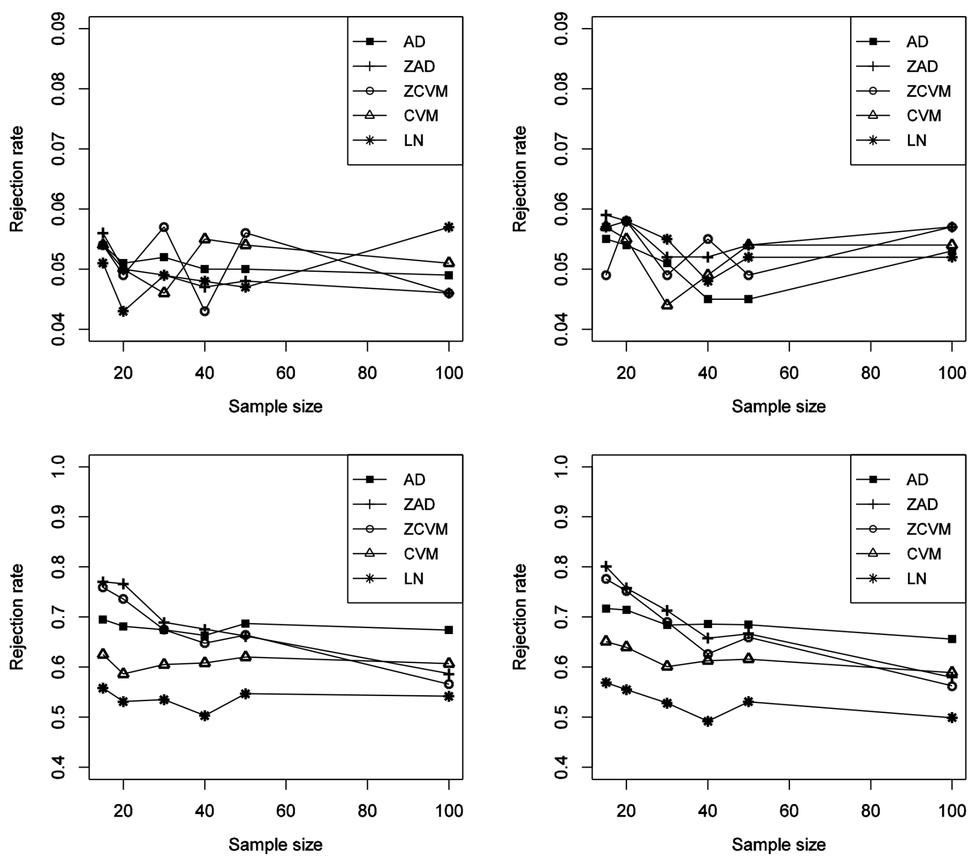

Fig.2: The rejection rate of the GOF tests for Fréchet distribution with respect to the sample of size, n= 15, 20, 30, 40, 50 and 100 at significance level 0.05. The rejection rate for Fréchet (μ = 100, σ = 10 and ξ = 0.5) is on the top left, Fréchet (μ = 0, σ = 1 and ξ = 0.3) is on the top right; Gamma is on the bottom left and Weibull on the bottom right.

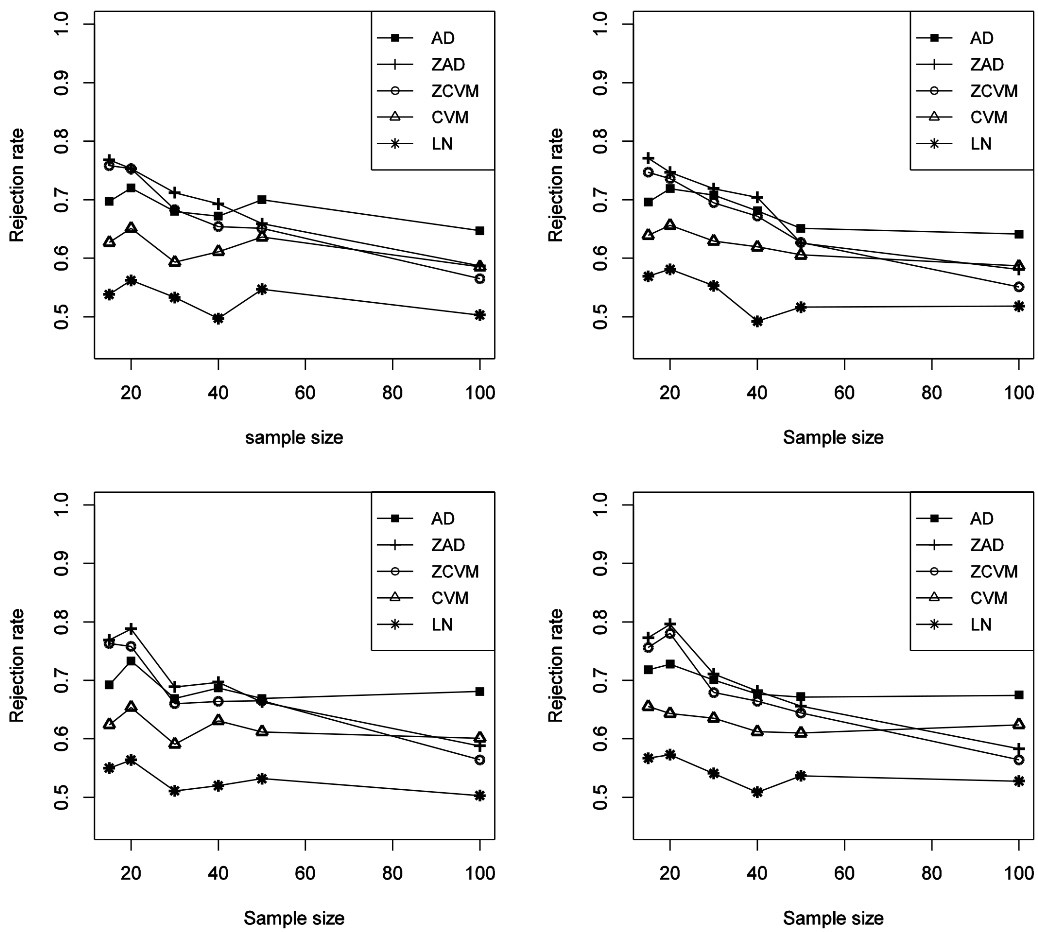

Fig.3: Rejection rate of the GOF tests for Fréchet distribution with respect to the sample of size n= 15, 20, 30, 40, 50 and 100 at significance level 0.05. The rejection rate for Lognormal is on the top left, Normal is on the top right; Generalized Logistics is on the bottom left and Exponential on the bottom right.

DISCUSSION

The assessment on the GOF tests begins with the development of critical values. The establishment of the critical values is crucial because they are the border points in deciding whether the selected statistical model fits the observed distribution or not. If the critical values are correct, the model will certainly match the observed distribution so the prediction of extreme event can be made effectively. Otherwise, the practitioners may deliver inaccurate information that will lead to the failure of prevention planning. Therefore, the validation of the critical values is crucial. For the purpose of validating the critical values, GOF for the true parameter values of the Fréchet distribution is evaluated first. Besides, it is important to determine whether the same result can be arrived at by using the same distribution with different parameters values. Despite having different parameters values, the GOF test should fail to reject H_0 because the distribution comes from the same parent distribution, which is the Fréchet distribution. The GOF test fails to reject H_0 if the rejection rates approximate the particular significance level.

For both Fréchet with true and different parameter values, the rejection rates of AD, ZAD, CVM, ZCVM and L_n tests are reliable because these rates are close to the respective significance level.

After the development and validation of critical values, the identification of the best GOF test is of interest. This is because the most powerful test yields the most accurate measurement on the degree of fit. In general, at significance level of 0.01, ZVCM is preferable for a small sample size (*n*=15 and 20), while the AD test performs the best for larger sample sizes (*n*=40, 50, 100). On the other hand, at 0.05 and 0.1 significance levels, ZAD is the most suitable test for GOF for small sample size (15 and 20) and sample of sizes *n*=30 and 40. For larger sample size (*n*=50 and 100), on the other hand, the AD critical values are the most powerful test.

CONCLUSION AND RECOMMENDATIONS

In this study, the performances of the GOF tests for Fréchet distribution were observed. Based on the parameter estimation by Maximum Likelihood, the ZAD and ZCVM tests are the most powerful ones for smaller sample sizes (ZAD for significance levels 0.05 and 0.1, ZCVM for significance level 0.01) as compared to AD which is more powerful for larger sample sizes. The GOF test for Fréchet can be assessed for different approaches of parameter estimations. In addition, the modification of the existing GOF can be extended to boost the power values. The sensitivity of the GOF tests can be evaluated in future studies.

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